Particle Spectra and Gauge Unification
in $SU(6) \times SU(2)_R$ Model

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Abstract

We study the path from the string scale physics to the low-energy physics in the $SU(6) \times SU(2)_R$ string-inspired model with the flavor symmetry $\mathbf{Z}_M \times \mathbf{Z}_N \times \tilde{D}_4$. The flavor symmetry controls the mass spectra of heavy particles as well as those of quarks and leptons in the intermediate energy region ranging from the string scale ($\sim 10^{18}$ GeV) to the electroweak scale. In this paper we examine the mass spectra of heavy particles in detail in our model. The renormalization group evolution of the gauge couplings is studied up to two-loop order. A consistent solution of the gauge unification around the string scale is found by adjusting the spectra of the anti-generation matter fields.

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1 Introduction

The study of the path for connecting the string theory with the low-energy physics is one of the most important issues in particle physics and cosmology. However, we are still in the early stages of the study. In fact, we do not yet have a comprehensive string-based understanding of apparent characteristic patterns in quark and lepton masses and mixings at low energies. It is considered that these characteristic patterns are closely linked to the flavor symmetry, which is expected to arise from the symmetric structure of the compact space in the string theory. It is also likely that the flavor symmetry controls the mass spectra of heavy particles in the intermediate energy region as well as those of quarks and leptons.

Recent developments in the string theory have provided new aspects of string phenomenology. It has been pointed out in Ref.[1] that a new type of non-Abelian flavor symmetry can appear additionally if the compact space is non-commutative. As a matter of fact, in a string with discrete torsion, the coordinates become non-commutative operators.[2, 3, 4] As a phenomenological approach, in Refs.[1] and [5] the flavor symmetry $\mathbf{Z}_M \times \mathbf{Z}_N \times \tilde{D}_4$ has been introduced into the string-inspired model, where the cyclic group $\mathbf{Z}_M$ and the binary dihedral group $\tilde{D}_4$ have R symmetries, while $\mathbf{Z}_N$ has not.[§] The introduction of a binary representation of non-Abelian flavor symmetry is motivated also by the phenomenological observation that the R-handed Majorana neutrino mass for the third generation is nearly equal to the geometrical average of the string scale M_S ($\sim 10^{18}$ GeV) and the electroweak scale M_Z. In Ref.[5], solving the anomaly-free conditions under many phenomenological constraints coming from the particle spectra, we were led to a large mixing angle (LMA)-MSW solution with $(M,\ N) = (19,\ 18)$, in which the appropriate flavor charges are assigned to the matter fields. The solution includes phenomenologically acceptable results concerning fermion masses and mixings and also concerning hierarchical energy scales including the string scale, μ scale and the Majorana mass scale of R-handed neutrinos.

In the framework of the string theory, we are prohibited from adding extra matter fields by hand. This situation is in sharp contrast to that of the conventional GUT-type models. In fact, in the perturbative heterotic string theory we have no adjoint or higher representation matter (Higgs) fields. Within this rigid framework, we have

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[§]We use here the notation "$\sim$" for the binary group. In Ref.[5] we have discussed the projective representation of the dihedral group D_4, which is identical with the binary dihedral group $\tilde{D}_4$.

discussed the path from the string scale physics to the low-energy physics in the $SU(6) \times SU(2)_R$ string-inspired model with the flavor symmetry $\mathbf{Z}_{19} \times \mathbf{Z}_{18} \times \tilde{D}_4$. In Ref.[6] the renormalization group (RG) equations down from the M_S have been studied for the scalar masses squared of the gauge non-singlet matter fields. It has been found that the radiative breaking of the gauge symmetry can occur slightly below the M_S due to the large Yukawa coupling which is identical with the colored Higgs coupling. This symmetry breaking triggers a subsequent symmetry breaking.[9] Then, we obtain the sequential symmetry breaking

$$SU(6) \times SU(2)_R \longrightarrow SU(4)_{\mathrm{PS}} \times SU(2)_L \times SU(2)_R \longrightarrow G_{\mathrm{SM}},$$

where $SU(4)_{\mathrm{PS}}$ and G_{SM} represent the Pati-Salam $SU(4)$[10] and the standard model gauge group, respectively.

The purpose of this paper is to pursue the further exploration of the path from the string scale physics to the low-energy physics. For this purpose we examine the particle spectra in the intermediate energy region in detail in the $SU(6) \times SU(2)_R$ model with the flavor symmetry $\mathbf{Z}_{19} \times \mathbf{Z}_{18} \times \tilde{D}_4$. Afterward, the RG runnings of the gauge couplings are studied up to two-loop order. It should be emphasized that in our model the unification scale is not the so-called GUT scale ($\sim 2 \times 10^{16}\,\mathrm{GeV}$) but around the string scale ($\sim 10^{18}\,\mathrm{GeV}$). We find a solution of the gauge coupling unification by adjusting the spectra of the anti-generation matter fields, with which the Kähler class moduli fields couple. In this solution $SU(3)_c$ and $SU(2)_L$ gauge couplings meet at $\varnothing(5 \times 10^{17}\,\mathrm{GeV})$. However, $SU(6)$ and $SU(2)_R$ gauge couplings are not perturbatively unified at the string scale. We expect that the non-perturbative unification of $SU(6)$ and $SU(2)_R$ gauge couplings is properly realized in the framework of the higher-dimensional underlying theory.

This paper is organized as follows. In §2, we explain main features of the $SU(6) \times SU(2)_R$ model with the flavor symmetry $\mathbf{Z}_{19} \times \mathbf{Z}_{18} \times \tilde{D}_4$. Although we have never yet found a concrete example of the string compactification which induces this type of the flavor symmetry exactly, $\mathbf{Z}_M \times \mathbf{Z}_N$ type of the flavor symmetry appears in some kinds of the Calabi-Yau compactification.[7, 8] Further, the binary type of the flavor symmetry is expected to stem from the compact space with non-commutative geometry. In this paper we assume $\mathbf{Z}_{19} \times \mathbf{Z}_{18} \times \tilde{D}_4$ as the flavor symmetry. In §3, we examine the particle spectra in the intermediate energy region ranging from the M_S to the M_Z. In the intermediate energy region there appear rich spectra of extra heavy particles beyond the minimal supersymmetric standard model. After

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presenting the two-loop RG equations of the gauge couplings in the intermediate region, we carry out the numerical analysis of the RG runnings of the gauge couplings in §4. We explore solutions of the gauge coupling unification. The final section is devoted to summary and discussion. In Appendix we explain dominant effective Yukawa couplings contributing to the RG evolution of the gauge couplings in the intermediate energy region.

2 $SU(6) \times SU(2)_R$ model with the flavor symmetry

Let us start by briefly summarizing the main points of the $SU(6) \times SU(2)_R$ string-inspired model considered here. More detailed descriptions are given in Refs.[1, 5, 6, 11, 12, 13, 14, 15].

(i). The gauge group is assumed to be $G = SU(6) \times SU(2)_R$, which can be obtained from E_6 through the $\mathbf{Z}_2$ flux breaking on a multiply-connected manifold K.[16, 17, 18] In contrast to the conventional GUT-type models, we have no Higgs fields of adjoint or higher representations. Nevertheless, the symmetry breaking of G down to G_{SM} can take place via the Higgs mechanism.[19]

(ii). In the context of the string theory, it is assumed that the gauge non-singlet matter content consists of the chiral superfields of three families and the single vector-like multiplet in the form

$$3 \times \mathbf{27}(\Phi_{1,2,3}) + (\mathbf{27}(\Phi_0) + \overline{\mathbf{27}}(\overline{\Phi})) \tag{1}$$

in terms of E_6. The superfields Φ in $\mathbf{27}$ of E_6 are decomposed into two irreducible representations of $G = SU(6) \times SU(2)_R$ as

$$\Phi(\mathbf{27}) = \begin{cases} \phi(\mathbf{15}, \mathbf{1}) & : & Q, L, g, g^c, S, \\ \psi(\mathbf{6}^*, \mathbf{2}) & : & (U^c, D^c), (N^c, E^c), (H_u, H_d), \end{cases} \tag{2}$$

where the pair g and g^c and the pair H_u and H_d represent the colored Higgs and the doublet Higgs superfields, respectively, N^c is the R-handed neutrino superfield, and S is an $SO(10)$ singlet. It should be noted that the doublet Higgs

and the color-triplet Higgs fields belong to different irreducible representations of G, as shown in Eq. (2). As a consequence, the triplet-doublet splitting problem is solved naturally.[11] In our model there are only two types of gauge invariant trilinear combinations

$$
\begin{aligned}
(\phi(\mathbf{15},\mathbf{1}))^3 &= QQg + Qg^cL + g^cgS, &\text{(3)} \\
\phi(\mathbf{15},\mathbf{1})(\psi(\mathbf{6^*},\mathbf{2}))^2 &= QH_dD^c + QH_uU^c + LH_dE^c + LH_uN^c \\
&\quad + SH_uH_d + gN^cD^c + gE^cU^c + g^cU^cD^c. &\text{(4)}
\end{aligned}
$$

(iii). As the flavor symmetry, we introduce the $\mathbf{Z}_{19} \times \mathbf{Z}_{18}$ and $\tilde{D}_4$ symmetries and regard $\mathbf{Z}_{19}$ and $\mathbf{Z}_{18}$ as the R and non-R symmetries, respectively. $\tilde{D}_4$ represents the binary dihedral group. Because the numbers 19 and 18 are relatively prime, we can combine these symmetries as

$$
\mathbf{Z}_{19} \times \mathbf{Z}_{18} = \mathbf{Z}_{342}. \tag{5}
$$

Solving the anomaly-free conditions under the many phenomenological constraints coming from the particle spectra, we obtain a LMA-MSW solution with the $\mathbf{Z}_{342}$ charges of the matter superfields, as shown in Table 1.[5] In this solution we assign the charge $(-1,\ 0)$ under $\mathbf{Z}_{19} \times \mathbf{Z}_{18}$, $i.e.$, the charge 18 under $\mathbf{Z}_{342}$ to the Grassmann number θ. The assignment of the "$\tilde{D}_4$ charges" to the matter superfields is given in Table 2, where σ_i $(i = 1, 2, 3)$ represent the Pauli matrices and

$$
\sigma_4 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \tag{6}
$$

The σ_3 transformation yields the R-parity. It is found that the R-parities of the superfields Φ_i $(i = 1, 2, 3)$ for the three generations are all odd, while those of Φ_0 and $\overline{\Phi}$ are even.

Due to the gauge symmetry and the flavor symmetry, the superpotential terms which induce the effective Yukawa couplings at low energies take the forms

$$
W_Y = \frac{1}{3!} z_0 \left(\frac{\phi_0 \overline{\phi}}{M_1^2} \right)^{\zeta_0} (\phi_0)^3 + \frac{1}{3!} \overline{z} \left(\frac{\phi_0 \overline{\phi}}{M_1^2} \right)^{\overline{\zeta}} (\overline{\phi})^3 + \frac{1}{2} h_0 \left(\frac{\phi_0 \overline{\phi}}{M_1^2} \right)^{\eta_0} \phi_0 \psi_0 \psi_0
$$

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Table 1: Assignment of $\mathbf{Z}_{342}$ charges for matter superfields. θ has the charge 18.

	Φ_1	Φ_2	Φ_3	Φ_0	$\overline{\Phi}$
$\phi(\mathbf{15},\ \mathbf{1})$	$a_1 = 126$	$a_2 = 102$	$a_3 = 46$	$a_0 = 12$	$\overline{a} = -16$
$\psi(\mathbf{6}^*,\ \mathbf{2})$	$b_1 = 120$	$b_2 = 80$	$b_3 = 16$	$b_0 = -14$	$\overline{b} = -67$

Table 2: Assignment of "$\tilde{D}_4$ charges" to matter superfields. The "$\tilde{D}_4$ charge" of θ is σ_1.

	$\Phi_i\ (i = 1, 2, 3)$	Φ_0	$\overline{\Phi}$
$\phi(\mathbf{15},\ \mathbf{1})$	σ_1	1	1
$\psi(\mathbf{6}^*,\ \mathbf{2})$	σ_2	σ_3	σ_4

$$+ \frac{1}{2}\,\overline{h}\left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\overline{\eta}}\left(\frac{\psi_0\overline{\psi}}{M_2^2}\right)^2 \overline{\phi}\,\overline{\psi}\,\overline{\psi} + \frac{1}{2}\sum_{i,j=1}^{3} z_{ij}\left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\zeta_{ij}}\phi_0\phi_i\phi_j$$

$$+ \frac{1}{2}\sum_{i,j=1}^{3} h_{ij}\left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\eta_{ij}}\phi_0\psi_i\psi_j + \sum_{i,j=1}^{3} m_{ij}\left(\frac{\phi_0\overline{\phi}}{M_1^2}\right)^{\mu_{ij}}\psi_0\phi_i\psi_j, \tag{7}$$

where $\phi_0\overline{\phi}$ and $\psi_0\overline{\psi}$ stand for the gauge-singlet combinations. The scale M_1 (M_2) represents the string scale M_S multiplied by the $\emptyset(1)$ factor coming from the volume of the compact space in which the matter fields $\phi_0(\mathbf{15},\ \mathbf{1})$ and $\overline{\phi}(\mathbf{15}^*,\ \mathbf{1})$ ($\psi_0(\mathbf{6}^*,\ \mathbf{2})$ and $\overline{\psi}(\mathbf{6},\ \mathbf{2})$) reside. The exponents are determined by the constraints coming from the flavor symmetry and concretely given by

$$(\zeta_0,\ \overline{\zeta},\ \eta_0,\ \overline{\eta}) \;=\; (0,\ 150,\ 158,\ 84), \qquad \zeta_{ij} = \begin{pmatrix} 57 & 51 & 37 \\ 51 & 45 & 31 \\ 37 & 31 & 17 \end{pmatrix}_{ij},$$

$$\eta_{ij} \;=\; \begin{pmatrix} 54 & 44 & 28 \\ 44 & 34 & 18 \\ 28 & 18 & 2 \end{pmatrix}_{ij}, \qquad \mu_{ij} = \begin{pmatrix} 49 & 39 & 23 \\ 43 & 33 & 17 \\ 29 & 19 & 3 \end{pmatrix}_{ij}. \tag{8}$$

The coefficients z_0, $\overline{z}$, h_0, $\overline{h}$, z_{ij}, h_{ij} and m_{ij} are $\emptyset(1)$ constants. The relation $\zeta_0 = 0$ means that only the superfield ϕ_0 takes part in the renormalizable interaction with the large Yukawa coupling at M_S. All or some of the powers of the gauge-singlet combination $\phi_0\overline{\phi}$ can be replaced with those of another gauge-singlet combination $\psi_0\overline{\psi}$ subject to the flavor symmetry.

Here it is important for us to comment on the role of the moduli fields. In the Calabi-Yau string, the generation matter and the anti-generation matter couple separately with the complex structure moduli fields and the Kähler class moduli fields, respectively, in the superpotential.[20] The nonvanishing vacuum expectation values (VEVs) of the moduli fields are expected to be $\emptyset(M_S)$. The VEVs of the Kähler class moduli fields represent the size and shapes of the compact space. In addition, the Kähler class moduli fields carry the flavor charges and their VEVs control the flavor symmetry. Therefore, there is a possibility that the second and the fourth terms in Eq. (7) are supplemented with the other terms multiplied by a certain function of the Kähler class moduli fields. In the next section we discuss the possible modification of the second term.

When ϕ_0 and $\overline{\phi}$ develop non-zero VEVs, the above non-renormalizable terms induce effective Yukawa couplings with hierarchical patterns. Namely, below the scale $|\langle\phi_0\rangle|$, the Froggatt-Nielsen mechanism acts for non-renormalizable interactions.[21] in the superpotential Further, we have another non-renormalizable terms

$$
W_1 = M_1^3 \left[l_0 \left(\frac{\phi_0\overline{\phi}}{M_1^2} \right)^{2n} + l_1 \left(\frac{\phi_0\overline{\phi}}{M_1^2} \right)^{n} \left(\frac{\psi_0\overline{\psi}}{M_2^2} \right)^{m} + l_2 \left(\frac{\psi_0\overline{\psi}}{M_2^2} \right)^{2m} \right] \tag{9}
$$

with $l_i = \emptyset(1)$. The flavor symmetry $\mathbf{Z}_{342} \times \tilde{D}_4$ requires $n = 81$ and $m = 4$. We assume that the supersymmetry is broken at the string scale due to the hidden sector dynamics and that the supersymmetry (SUSY) breaking is communicated gravitationally to the observable sector via the universal soft SUSY breaking terms. The scale of the SUSY breaking $\tilde{m}_\phi$ is supposed to be 10^3 GeV. Under this assumption we study the minimum point of the scalar potential.[9] When we have a large coupling for the Yukawa interaction $z_0 \phi_0^3$ at the string scale M_S, through the RG evolution the scalar masses squared become negative slightly below M_S.[6] As a consequence,

the ground state is characterized by $\langle \phi_i \rangle = \langle \psi_i \rangle = 0$, $(i = 1, 2, 3)$ and

$$\frac{|\langle \phi_0 \rangle|}{M_1} = \frac{|\langle \overline{\phi} \rangle|}{M_1} = 0.894, \qquad \frac{|\langle \psi_0 \rangle|}{M_2} = \frac{|\langle \overline{\psi} \rangle|}{M_2} = 0.103, \tag{10}$$

where we take the numerical values $M_1 = M_2 = 5 \times 10^{17}\,\mathrm{GeV} = M_S/3$ and

$$\left(\frac{|\langle \phi_0 \rangle|}{M_1} \right)^{4n-2} \times M_1 = c\,\tilde{m}_\phi. \tag{11}$$

The coefficient c in the r. h. s. is expressed as a function of $l_{0,1,2}$, n and m and we take here $c = 0.1$. In this vacuum we have the relations

$$x^{81} \simeq y^4, \qquad x^{161}\,M_1 = 10^2\,\mathrm{GeV}, \tag{12}$$

where we use the notation $x = (|\langle \phi_0 \rangle|/M_1)^2$ and $y = (|\langle \psi_0 \rangle|/M_2)^2$. Thus, below the scale $|\langle \psi_0 \rangle|$ we obtain the effective Yukawa superpotential

$$\begin{aligned}
W_Y^{(eff)} &= \frac{1}{3!}\,z_0\,x^{\zeta_0}(\phi_0)^3 + \frac{1}{3!}\,\overline{z}\,x^{\overline{\zeta}}(\overline{\phi})^3 + \frac{1}{2}\,h_0\,x^{\eta_0}\,\phi_0\psi_0\psi_0 \\[2mm]
&\quad + \frac{1}{2}\,\overline{h}\,x^{\overline{\eta}}\,y^2\overline{\phi}\,\overline{\psi}\,\overline{\psi} + \frac{1}{2}\sum_{i,j=1}^{3} z_{ij}\,x^{\zeta_{ij}}\,\phi_0\phi_i\phi_j \\[2mm]
&\quad + \frac{1}{2}\sum_{i,j=1}^{3} h_{ij}\,x^{\eta_{ij}}\,\phi_0\psi_i\psi_j + \sum_{i,j=1}^{3} m_{ij}\,x^{\mu_{ij}}\,\psi_0\phi_i\psi_j.
\end{aligned} \tag{13}$$

3   Mass spectra of heavy particles

In this section we explore the particle spectra in the intermediate energy region ranging from the M_S to the M_Z for the R-parity even superfields first and then for the odd superfields.

A. The R-parity even superfields

The R-parity even superfields contain ϕ_0, $\overline{\phi}$, ψ_0 and $\overline{\psi}$. As mentioned above, the gauge symmetry is spontaneously broken at the scale $|\langle \phi_0(\mathbf{15},\ \mathbf{1})\rangle| \simeq 4.5 \times 10^{17}\,\mathrm{GeV}$, and subsequently at the scale $|\langle \psi_0(\mathbf{6^*},\ \mathbf{2})\rangle| \simeq 5.2 \times 10^{16}\,\mathrm{GeV}$. This yields the symmetry breakings

$$SU(6) \times SU(2)_R \xrightarrow{\langle \phi_0 \rangle} SU(4)_{\mathrm{PS}} \times SU(2)_L \times SU(2)_R \xrightarrow{\langle \psi_0 \rangle} G_{\mathrm{SM}}. \tag{14}$$

Since the fields that develop non-zero VEVs are singlets under the remaining gauge symmetries, they are assigned as $\langle \phi_0(\mathbf{15},\ \mathbf{1})\rangle = \langle S_0 \rangle$ and $\langle \psi_0(\mathbf{6^*},\ \mathbf{2})\rangle = \langle N_0^c \rangle$. In the first step of the symmetry breaking, the fields Q_0, L_0, $\overline{Q}$, $\overline{L}$ and $(S_0 - \overline{S})/\sqrt{2}$ are absorbed by the gauge fields. Through the subsequent symmetry breaking, the fields U_0^c, E_0^c, $\overline{U}^c$, $\overline{E}^c$ and $(N_0^c - \overline{N}^c)/\sqrt{2}$ are absorbed. Therefore, below the scale $|\langle \psi_0 \rangle|$ the remaining modes in the R-parity even superfields are gauge singlet fields $(S' \equiv (S_0 + \overline{S})/\sqrt{2},\ N'^c \equiv (N_0^c + \overline{N}^c)/\sqrt{2})$, doublet Higgs fields $(H_{u_0}, H_{d_0}, \overline{H}_u, \overline{H}_d)$ and down-type colored fields $(g_0, g_0^c, D_0^c, \overline{g}, \overline{g}^c, \overline{D}^c)$.

(i). Gauge singlet fields S' and N'^c

Mass matrix for S' and N'^c is induced from Eq. (9) as

$$\widehat{\mathcal{M}}_{S'N'} = \begin{array}{c} \\ S' \\ N'^c \end{array}\!\!\begin{array}{c} S' \qquad\qquad\qquad\qquad N'^c \\ \left(\begin{array}{cc} l_0 \text{\O}(4n^2)x^{2n-1} & l_1 \text{\O}(nm)x^{2n(1-1/4m)-1/2} \\ l_1 \text{\O}(nm)x^{2n(1-1/4m)-1/2} & l_2 \text{\O}(4m^2))x^{2n(1-1/2m)} \end{array} \right), \end{array} \tag{15}$$

in M_1 units, where $n = 81$ and $m = 4$. The eigenvalues are given by

$$\text{\O}(x^{118}\, M_1) \sim 10^{6.2}\,\mathrm{GeV}, \qquad \text{\O}(x^{125}\, M_1) \sim 10^{5.5}\,\mathrm{GeV}. \tag{16}$$

In the following we use the notation $\tilde{S}$ and $\tilde{N}^c$ for the mass eigenstates.

(ii). Doublet Higgs fields H_{u_0}, H_{d_0}, $\overline{H}_u$ and $\overline{H}_d$

The mass matrix for these fields is derived from the terms in Eqs. (9) and (13). The effective superpotential contributing to this mass matrix becomes

$$W_H^{(eff)} \simeq h_0\, x^{\eta_0}\, S_0 H_{u0} H_{d0} + \overline{h}\, x^{\overline{\eta}}\, y^2\, \overline{SH_u H_d} + l_1\, x^n\, y^{m-1}\, (H_{u_0}\overline{H}_u + H_{d_0}\overline{H}_d), \tag{17}$$

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where the first term induces the so-called μ-term with $\mu = \emptyset(x^{\eta_0+1/2} M_1)$. Using $(\eta_0, \bar{\eta}) = (158, 84)$ and $x^{81} \simeq y^4$, we obtain the mass matrix

$$
\widehat{\mathcal{M}}_H \simeq
\begin{array}{c}
 \\
H_{u_0} \\
\overline{H_d}
\end{array}
\begin{array}{c}
\begin{array}{cc}
H_{d_0} & \overline{H_u}
\end{array} \\
\left(
\begin{array}{cc}
h_0\, x^{158.5} & l_1\, x^{141.75} \\
l_1\, x^{141.75} & \overline{h}\, x^{125}
\end{array}
\right)
\end{array}
\tag{18}
$$

in M_1 units. The eigenvalues are

$$
\emptyset(x^{125} M_1) \sim 10^{5.5}\,\text{GeV}, \qquad \emptyset(x^{158.5} M_1) \sim 10^{2.2}\,\text{GeV}. \tag{19}
$$

(iii). Down-type colored fields g_0, g_0^c, D_0^c, $\overline{g}$, $\overline{g}^c$ and $\overline{D}^c$

The effective superpotential which yields the mass matrix for these fields is of the form

$$
\begin{aligned}
W_g^{(eff)} \simeq\ & z_0\, x^{\zeta_0}\, S_0 g_0 g_0^c + \overline{z}\, x^{\overline{\zeta}}\, \overline{S}\,\overline{g}\,\overline{g}^c + h_0\, x^{\eta_0}\, g_0 N_0^c D_0^c + \overline{h}\, x^{\overline{\eta}}\, \overline{g}\,\overline{N^c}\overline{D^c} \\
& + l_0\, x^{2n-1}\, (g_0\overline{g} + g_0^c\overline{g^c}) + l_1\, x^n\, y^{m-1}\, D_0^c\overline{D^c}.
\end{aligned}
\tag{20}
$$

This effective superpotential leads to the mass matrix

$$
\widehat{\mathcal{M}}_g \simeq
\begin{array}{c}
 \\
g_0 \\
\overline{g}^c \\
\overline{D}^c
\end{array}
\begin{array}{c}
\begin{array}{ccc}
g_0^c & \overline{g} & D_0^c
\end{array} \\
\left(
\begin{array}{ccc}
z_0\, x^{0.5} & l_0\, x^{161} & h_0\, x^{168.125} \\
l_0\, x^{161} & \overline{z}\, x^{150.5} & 0 \\
0 & \overline{h}\, x^{134.625} & l_1\, x^{141.75}
\end{array}
\right)
\end{array}
\tag{21}
$$

in M_1 units. Here, (2, 3) and (3, 1) elements in this matrix are not exactly zero but approximately zero. In fact, the terms $\overline{S}N_0^c\overline{g^c}D_0^c$ and $S_0\overline{N_0^c}g_0^c\overline{D^c}$ are induced through the higher order effects and sufficiently suppressed compared to the terms in Eq. (20). The eigenvalues of this matrix are $\emptyset(x^{0.5} M_1)$, $\emptyset(x^{134.625} M_1)$ and $\emptyset(x^{157.625} M_1)$. So it turns out that one set of down-type colored super-fields with even R-parity should exist around $x^{157.625} M_1 \simeq \emptyset(10^{2.3}\,\text{GeV})$. As pointed out in the previous section, however, in the Calabi-Yau string the anti-generation matter fields couple with the Kähler class moduli fields, which carries

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the flavor charge. Therefore, both the second and the fourth terms in Eq. (13), i.e., $\overline{\phi}^3$ and $\overline{\phi\psi}^2$ terms, are possibly supplemented with the other terms. Since we have never yet known the definite content of the Kähler class moduli fields and also their flavor charges, the explicit form of the interactions of the Kähler class moduli fields is undetermined in the present approach. In this paper, from the phenomenological viewpoint, we consider a simple case that one of the Kähler class moduli fields couple with the anti-generation fields $\overline{\phi}$ only via the interaction

$$f(T) \left(\frac{S_0 \overline{S}}{M_1^2} \right)^{\overline{k}} \overline{\phi}^3, \tag{22}$$

where T is one of the Kähler class moduli fields and carries an appropriate flavor charge. The exponent $\overline{k}$ is introduced as an unknown parameter, because we can not determine the flavor charge of T at this stage as well as the explicit form of the function $f(T)$. In view of the fact that the Kähler class moduli fields represent the size and the shape of the compact space, it is supposed that the field T develops a non-zero VEV with $\langle T \rangle = \emptyset(M_S)$ and $f(\langle T \rangle) = \emptyset(1)$. If $\overline{k} < 150.5$, a dominant term contributing to the $(2, 2)$ element in Eq. (21) is replaced by

$$f(\langle T \rangle)\, x^{\overline{k}}\, \langle \overline{S} \rangle \overline{g}\overline{g^c} = M_1\, f(\langle T \rangle)\, x^k\, \overline{g}\overline{g^c}, \tag{23}$$

where $k = \overline{k} + 0.5$. Then the mass matrix is also replaced by

$$\widehat{\mathcal{M}}_g \simeq \begin{array}{c} \\ g_0 \\ \overline{g^c} \\ \overline{D^c} \end{array} \begin{array}{ccc} g_0^c & \overline{g} & D_0^c \\ \left(\begin{array}{ccc} z_0\, x^{0.5} & l_0\, x^{161} & h_0\, x^{168.125} \\ l_0\, x^{161} & f(\langle T \rangle)\, x^k & 0 \\ 0 & \overline{h}\, x^{134.625} & l_1\, x^{141.75} \end{array} \right) \end{array} \tag{24}$$

in M_1 units. If we take $10.125 < k < 134.625$, the eigenvalues of this matrix become

$$\emptyset(x^{0.5}\, M_1) \sim 10^{17.7}\, \mathrm{GeV}, \quad \emptyset(x^k\, M_1), \quad \emptyset(x^{141.75}\, M_1) \sim 10^{3.9}\, \mathrm{GeV} \tag{25}$$

with $x^{10.125}\, M_1 \sim 10^{16.7}\, \mathrm{GeV} > x^k\, M_1 > x^{134.625}\, M_1 \sim 10^{4.6}\, \mathrm{GeV}$. In the next section we adjust the parameter k so as to find a consistent solution of gauge

Table 3: The multiplicities and the spectra of the R-parity even superfields. In this table the parameter k is supposed to be $10.125 < k < 118$, because k is adjusted as $k = 42.5$ later.

	$\times M_1$		Q	L	U^c	E^c	(H_u, H_d)	(g, g^c, D^c)	(S, N^c)
1	$\sim$	$x^{0.5}$	2	2	2	2	4	6	4
$x^{0.5}$	$\sim$	$x^{10.125}$	0	0	2	2	4	4	3
$x^{10.125}$	$\sim$	x^{k}	0	0	0	0	4	4	2
x^{k}	$\sim$	x^{118}	0	0	0	0	4	2	2
x^{118}	$\sim$	x^{125}	0	0	0	0	4	2	1
x^{125}	$\sim$	$x^{141.75}$	0	0	0	0	2	2	0
$x^{141.75}$	$\sim$	$x^{158.5}$	0	0	0	0	2	0	0
$x^{158.5}$	$\sim$		0	0	0	0	0	0	0

unification. Thus, the Kähler class moduli fields, although are gauge-singlets, possibly affect the evolution of the gauge couplings through the couplings with the anti-generation fields.

We summarize the multiplicities and the spectra of the R-parity even superfields in Table 3. We suppose here $10.125 < k < 118$, because the parameter k is adjusted as $k = 42.5$ later.

B. The R-parity odd superfields

The R-parity odd superfields contains three generations of matter superfields ϕ_i and ψ_i $(i = 1, 2, 3)$. We study here the mass spectra of these fields in order.

(i). Up-type quarks

As seen from the last term in Eq. (13) which contains $m_{ij} x^{\mu_{ij}} Q_i U_j^c H_{u0}$, the mass matrix for up-type quarks becomes

$$\mathcal{M}_{ij} v_u = m_{ij}\, x^{\mu_{ij}} v_u\,, \tag{26}$$

where $v_u = \langle H_{u0} \rangle$ and $m_{ij} = \emptyset(1)$. The exponents μ_{ij} are given in Eq. (8). The eigenvalues of the mass matrix are

$$\emptyset(x^{49}\, v_u), \qquad \emptyset(x^{33}\, v_u), \qquad \emptyset(x^3\, v_u), \tag{27}$$

which correspond to m_u, m_c and m_t, respectively. Since $v_u = \emptyset(10^2\,\mathrm{GeV})$, all of up-type quarks remain massless in the intermediate energy region.

(ii). Down-type colored fields

The effective superpotential of down-type colored fields with odd R-parity takes the form

$$W_D^{(eff)} = \sum_{i,j=1}^{3} \left(z_{ij}\, x^{\zeta_{ij}}\, S_0 g_i g_j^c + m_{ij}\, x^{\mu_{ij}}\, N_0^c g_i D_j^c + m_{ij}\, x^{\mu_{ij}}\, H_{d0} Q_i D_j^c \right). \tag{28}$$

When S_0, N_0^c and H_{d0} develop nonvanishing VEVs, the mass matrix is derived as

$$\widehat{\mathcal{M}}_d = \begin{array}{c} \\ g \\ D \end{array} \overset{\begin{array}{cc} g^c & D^c \end{array}}{\left(\begin{array}{cc} x^{0.5}\mathcal{Z} & y^{0.5}\mathcal{M} \\ 0 & \rho_d \mathcal{M} \end{array} \right)} \tag{29}$$

in M_1 units. The submatrices $\mathcal{Z}$ and $\mathcal{M}$ are given by

$$\mathcal{Z}_{ij} = z_{ij}\, x^{\zeta_{ij}} \tag{30}$$

and Eq. (26), respectively and $\rho_d = v_d/M_1$ with $v_d = \langle H_{d0} \rangle$. The mass matrix $\widehat{\mathcal{M}}_d$ yields mixings between g^c and D^c and has six eigenvalues. Three of them represent light modes. Solving the eigenvalue problem of this matrix, we obtain their masses[1, 11, 14]

$$\emptyset(x^{49}\, v_d), \qquad \emptyset(x^{41}\, v_d), \qquad \emptyset(x^{21}\, v_d), \tag{31}$$

which correspond to m_d, m_s and m_b, respectively. The remaining three represent heavy modes with their masses

$$\emptyset(x^{13.125}\, M_1), \qquad \emptyset(x^{31.5}\, M_1), \qquad \emptyset(x^{49.125}\, M_1). \tag{32}$$

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(iii). Charged leptons and extra charged Higgs fields with odd R-parity

For these fields the effective superpotential is written as

$$W_E^{(eff)} = \sum_{i,j=1}^{3} \left(h_{ij}\, x^{\eta_{ij}}\, S_0 H_{di} H_{uj} + m_{ij}\, x^{\mu_{ij}}\, N_0^c L_i H_{uj} + m_{ij}\, x^{\mu_{ij}}\, H_{d0} L_i E_j^c \right).$$

(33)

The mass matrix is similar to that for the down-type colored fields. Concretely, we have

$$\widehat{\mathcal{M}}_l = \begin{array}{c} \\ H_d \\ L \end{array} \begin{array}{c} H_u \qquad E^c \\ \left(\begin{array}{cc} x^{0.5}\mathcal{H} & 0 \\ y^{0.5}\mathcal{M} & \rho_d \mathcal{M} \end{array} \right) \end{array}$$

(34)

in M_1 units, where the submatrix $\mathcal{H}$ is given by

$$\mathcal{H}_{ij} = h_{ij}\, x^{\eta_{ij}}.$$

(35)

The mass matrix $\widehat{\mathcal{M}}_l$ yields L-H_d mixings. Among six eigenvalues of $\widehat{\mathcal{M}}_l$, three represent light modes corresponding to charged leptons. Their masses are given by[1, 11, 15]

$$\varnothing(x^{49}\, v_d), \qquad \varnothing(x^{35}\, v_d), \qquad \varnothing(x^{17}\, v_d),$$

(36)

which represent m_e, m_μ and m_τ, respectively. The remaining three represent heavy modes with their masses

$$\varnothing(x^{2.5}\, M_1), \qquad \varnothing(x^{29.125}\, M_1), \qquad \varnothing(x^{44.5}\, M_1).$$

(37)

(iv). Neutral fields with odd R-parity

The neutral sector contains five types of matter fields, H_u^0, H_d^0, L^0, N^c and S in our model. Then we have the 15×15 mass matrix[1, 11, 12, 13, 15]

$$\widehat{\mathcal{M}}_{NS} = \begin{array}{c} \\ H_u^0 \\ H_d^0 \\ L^0 \\ N^c \\ S \end{array} \begin{array}{c} H_u^0 \qquad H_d^0 \qquad L^0 \qquad N^c \qquad S \\ \left(\begin{array}{ccccc} 0 & x^{0.5}\mathcal{H} & y^{0.5}\mathcal{M}^T & 0 & \rho_d \mathcal{M}^T \\ x^{0.5}\mathcal{H} & 0 & 0 & 0 & \rho_u \mathcal{M}^T \\ y^{0.5}\mathcal{M} & 0 & 0 & \rho_u \mathcal{M} & 0 \\ 0 & 0 & \rho_u \mathcal{M}^T & \mathcal{N} & 0 \\ \rho_d \mathcal{M} & \rho_u \mathcal{M} & 0 & 0 & \mathcal{S} \end{array} \right) \end{array}$$

(38)

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in M_S units, where $\rho_u = v_u/M_1$. In this mass matrix, the 3×3 submatrix $\mathcal{N}$ plays the role of the R-handed Majorana mass matrix in the seesaw mechanism. Among fifteen eigenvalues of $\widehat{\mathcal{M}}_{NS}$ three represent tiny masses of neutrinos,[1, 15] which are given by

$$\frac{v_u^2}{M_1 \, x^{37.25}} \times (\varnothing(x^{12}), \quad \varnothing(x^4), \quad \varnothing(1) \,). \tag{39}$$

Six of them are degenerate with the above-mentioned three heavy modes coming from extra charged Higgs fields. The remaining six eigenvalues are approximately given by the submatrices $\mathcal{N}$ and $\mathcal{S}$. $\mathcal{N}$ is induced from the term

$$\frac{n_{ij}}{M_1} \, x^{\nu_{ij}} \, (N_i^c \overline{N^c})(N_j^c \overline{N^c}) \tag{40}$$

and of the form

$$\mathcal{N}_{ij} = n_{ij} \, x^{\nu_{ij}} \, y \tag{41}$$

in M_1 units, where the flavor symmetry leads to the relation $\nu_{ij} = \eta_{ij} + 49$. The three eigenvalues of $\mathcal{N}$ become

$$\varnothing(x^{123.25} \, M_1), \qquad \varnothing(x^{103.25} \, M_1), \qquad \varnothing(x^{71.25} \, M_1), \tag{42}$$

which represent the masses of the R-handed Majorana neutrinos. The largest eigenvalue $\varnothing(x^{71.25} \, M_1) \simeq 10^{10.8}$ GeV is nearly equal to the geometrical average of M_S and M_Z. The submatrix $\mathcal{S}$ induced from

$$\frac{s_{ij}}{M_1} \, x^{\sigma_{ij}} \, (S_i \overline{S})(S_j \overline{S}) \tag{43}$$

is given by

$$\mathcal{S}_{ij} = s_{ij} \, x^{\sigma_{ij}+1} \tag{44}$$

in M_1 units, where we have the relation $\sigma_{ij} = \zeta_{ij} - 11$. The eigenvalues of $\mathcal{S}$ become

$$\varnothing(x^{47} \, M_1), \qquad \varnothing(x^{35} \, M_1), \qquad \varnothing(x^7 \, M_1). \tag{45}$$

In Table 4 we summarize the multiplicities of the R-parity odd superfields. It is worthy to note that in the intermediate energy region we have the hierarchical spectra of heavy particles including the R-handed Majorana neutrinos.

Table 4: The multiplicities and the spectra of the R-parity odd superfields

$\times M_1$			Q	U^c	g	(D^c, g^c)	E^c	H_u	(L, H_d)	N^c	S
1	$\sim$	$x^{2.5}$	3	3	3	6	3	3	6	3	3
$x^{2.5}$	$\sim$	x^7	3	3	3	6	3	2	5	3	3
x^7	$\sim$	$x^{13.125}$	3	3	3	6	3	2	5	3	2
$x^{13.125}$	$\sim$	$x^{29.125}$	3	3	2	5	3	2	5	3	2
$x^{29.125}$	$\sim$	$x^{31.5}$	3	3	2	5	3	1	4	3	2
$x^{31.5}$	$\sim$	x^{35}	3	3	1	4	3	1	4	3	2
x^{35}	$\sim$	$x^{44.5}$	3	3	1	4	3	1	4	3	1
$x^{44.5}$	$\sim$	x^{47}	3	3	1	4	3	0	3	3	1
x^{47}	$\sim$	$x^{49.125}$	3	3	1	4	3	0	3	3	0
$x^{49.125}$	$\sim$	$x^{71.25}$	3	3	0	3	3	0	3	3	0
$x^{71.25}$	$\sim$	$x^{103.25}$	3	3	0	3	3	0	3	2	0
$x^{103.25}$	$\sim$	$x^{123.25}$	3	3	0	3	3	0	3	1	0
$x^{123.25}$	$\sim$		3	3	0	3	3	0	3	0	0

4 RG equations and numerical study

Now we study the two-loop RG evolutions of the gauge couplings in our model. The evolution equations up to the two-loop order for $\alpha_i = g_i^2/4\pi$ are generally given by[22]

$$\frac{d\alpha_i}{dt} = \frac{1}{2\pi}\left[-b_i + \frac{1}{4\pi}\left(\sum_j b_{ij}\alpha_j - a_i\right)\right]\alpha_i^2, \qquad (46)$$

where $t = \ln(Q/Q_0)$ with $Q_0 = M_S$. The subscripts i and j specify the gauge group. The coefficients b_i, b_{ij} and a_i are determined depending on the particle contents and their spectra. The a_i terms represent the contributions of Yukawa interactions.

In our model, the values of the gauge couplings g_6 of $SU(6)$ and g_{2R} of $SU(2)_R$ are introduced at the string scale M_S as an initial condition. As mentioned above, the gauge group $SU(6) \times SU(2)_R$ is broken down to $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ at the energy scale $|\langle\phi_0\rangle| = x^{0.5}\, M_1$. The subsequent breaking of the gauge group into

$SU(3)_c \times SU(2)_L \times U(1)_Y$ occurs at the energy scale $|\langle \psi_0 \rangle| = y^{0.5} M_1 \simeq x^{10.125} M_1$. The supersymmetry is supposed to be broken at the scale $\tilde{m}_\phi = 10^3 \, \text{GeV} = x^{151} M_1$. From the particle spectra given in Tables 3 and 4, we can determine the coefficients b_i and b_{ij} in various energy regions ranging from M_S to M_Z.

In the first region between M_S $(t = 0)$ and $x^{0.5} M_1$ $(t = -1.21)$, where the gauge symmetry is $SU(6) \times SU(2)_R$, we have

$$- b_i = \begin{pmatrix} -3 \\ 9 \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} 209 & 15 \\ 175 & 81 \end{pmatrix}, \quad a_i = \begin{pmatrix} 8 \\ 0 \end{pmatrix} y_0 \tag{47}$$

with $y_0 = |z_0|^2/4\pi$. As seen from Eqs. (7) and (8), in this region only $z_0(\phi_0)^3$ term contributes to the RG equations for gauge couplings at two-loop level. The subscripts $i, j = 6$ and $2R$ denote $SU(6)$ and $SU(2)_R$, respectively. The one-loop RG equation for the Yukawa coupling y_0 is given by

$$\frac{dy_0}{dt} = \frac{1}{2\pi} \left(-28 \, \alpha_6 + 9 \, y_0 \right) y_0. \tag{48}$$

As will be discussed later, the evolution of the gauge couplings are rather insensitive to the Yukawa couplings in the whole region. Then, in the present analysis it is sufficient for us to take the one-loop RG equations for the Yukawa couplings into account.

In the second region between $x^{0.5} M_1$ $(t = -1.21)$ and $x^{10.125} M_1$ $(t = -3.37)$, the gauge group is $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$. In this region we obtain

$$- b_i = \begin{pmatrix} 0 \\ n_H \\ 4 + n_H \end{pmatrix}, \quad b_{ij} = \begin{pmatrix} 100 & 9 & 15 \\ 45 & 18 + 7n_H & 3n_H \\ 75 & 3n_H & 46 + 7n_H \end{pmatrix},$$

$$a_i = \begin{pmatrix} 4 \left(y^{(Z)} + 2y^{(1)} + 2y^{(2)} + 4y^{(3)} \right) \\ 4 \left(y^{(H)} + 4y^{(1)} + 4y^{(2)} + y^{(4)} \right) \\ 4 \left(y^{(H)} + 4y^{(1)} + 4y^{(2)} + 3y^{(3)} + y^{(4)} \right) \end{pmatrix}, \tag{49}$$

where $i, j = 4, 2L$ and $2R$ means $SU(4)_{PS}$, $SU(2)_L$ and $SU(2)_R$, respectively. The notation n_H in Eq. (49) represents the multiplicity of doublet Higgs fields and is

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given by

$$n_H = \begin{cases} 5 & x^{0.5} > Q/M_1 \geq x^{2.5}, \\ 4 & x^{2.5} > Q/M_1 \geq x^{10.125}. \end{cases} \tag{50}$$

At $x^{0.5} M_1$ we use the continuity condition

$$\alpha_6 = \alpha_4 = \alpha_{2L}. \tag{51}$$

The dominant contributions of the effective Yukawa interactions are of the forms

$$y^{(Z)} = \frac{1}{4\pi}|f_{33}^{(Z)}|^2, \qquad y^{(1)} = \frac{1}{4\pi}|M_{33}^{(1)}|^2, \qquad y^{(3)} = \frac{1}{4\pi}|M_{33}^{(3)}|^2,$$

$$y^{(H)} = \frac{1}{4\pi}|f_{33}^{(H)}|^2\, \theta\left(\frac{Q}{M_1} - x^{2.5}\right), \quad y^{(2)} = \frac{1}{4\pi}|M_{33}^{(2)}|^2\, \theta\left(\frac{Q}{M_1} - x^{2.5}\right),$$

$$y^{(4)} = \frac{1}{4\pi}|M_{33}^{(4)}|^2\, \theta\left(\frac{Q}{M_1} - x^{2.5}\right). \tag{52}$$

The definition of the effective Yukawa couplings $f_{33}^{(Z,H)}$ and $M_{33}^{(1\sim4)}$ are presented in the Appendix. The RG equations for the Yukawa couplings are also given in the Appendix.

In the third region ranging from $x^{10.125} M_1$ ($t = -3.37$) to $x^{151} M_1(= \tilde{m}_\phi = 10^3\,\text{GeV})$ ($t = -35.00$), where the gauge group coincides with $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$, still being supersymmetric, we obtain

$$-b_i = \begin{pmatrix} -3 + N_g \\ n_H \\ 6 + \frac{2}{5}N_g + \frac{3}{5}n_H \end{pmatrix},$$

$$b_{ij} = \begin{pmatrix} 14 & 9 & 11/5 \\ 24 & 18 & 6/5 \\ 88/5 & 18/5 & 38/5 \end{pmatrix} + \begin{pmatrix} 34/3 & 0 & 4/15 \\ 0 & 0 & 0 \\ 32/15 & 0 & 8/75 \end{pmatrix} N_g$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 7 & 3/5 \\ 0 & 9/5 & 9/25 \end{pmatrix} n_H,$$

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$$a_i = \begin{pmatrix} 4 \\ 6 \\ 26/5 \end{pmatrix} y^{(1U)} + \begin{pmatrix} 4 \\ 6 \\ 14/5 \end{pmatrix} y^{(1D)}$$

$$+ \begin{pmatrix} 1 \\ 0 \\ 2/5 \end{pmatrix} \left(y^{(3N)} + 2y^{(3D)} + 2y'^{(Z)} \right), \tag{53}$$

where $i, j = 3, 2L$ and 1 means $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, respectively. n_H and N_g, the latter being the multiplicity of the extra down-type colored fields, are shown in Table 5. The Yukawa terms are expressed as

$$y^{(1U)} = \frac{1}{4\pi}|M_{33}^{(1U)}|^2, \qquad y^{(1D)} = \frac{1}{4\pi}|M_{33}^{(1D)}|^2 \, \theta\left(\frac{Q}{M_1} - x^{13.125}\right),$$

$$y^{(3N)} = \frac{1}{4\pi}|M_{33}^{(3N)}|^2 \, \theta\left(\frac{Q}{M_1} - x^{13.125}\right), \quad y^{(3D)} = \frac{1}{4\pi}|M_{33}^{(3D)}|^2 \, \theta\left(\frac{Q}{M_1} - x^{13.125}\right),$$

$$y'^{(Z)} = \frac{1}{4\pi}|f_{33}'^{(Z)}|^2 \, \theta\left(\frac{Q}{M_1} - x^{13.125}\right). \tag{54}$$

The continuity conditions at the scale $x^{10.125} M_1$ are given by

$$\alpha_4 = \alpha_3, \tag{55}$$

$$\alpha_1^{-1} = \frac{2}{5}\alpha_4^{-1} + \frac{3}{5}\alpha_{2R}^{-1}, \tag{56}$$

$$y^{(1)} = y^{(1U)} = y^{(1D)}, \tag{57}$$

$$y^{(3)} = y^{(3N)} = y^{(3D)}, \tag{58}$$

$$f_{33}'^{(Z)} = x^{4.375} f_{33}^{(Z)} + \frac{1}{\sqrt{2}} M_{33}^{(3)}. \tag{59}$$

The last condition is due to the mixing of g^c and D^c as seen in Eq. (29). In the Appendix we list the definitions of $f_{33}'^{(Z)}$ and $M_{33}^{(1U \sim 3D)}$ and the RG equations for the Yukawa couplings.

Table 5: The values of n_H and N_g in the region between $x^{10.125} M_1$ and $x^{151} M_1$. n_H and N_g are the multiplicities of doublet Higgses and extra down-type colored fields, respectively. Here, the parameter k is taken as $31.5 < k < 44.5$. In the numerical calculation, we use the adjusted value $k = 42.5$.

$\times M_1$			n_H	N_g
$x^{10.125}$	$\sim$	$x^{13.125}$	4	5
$x^{13.125}$	$\sim$	$x^{29.125}$	4	4
$x^{29.125}$	$\sim$	$x^{31.5}$	3	4
$x^{31.5}$	$\sim$	x^{k}	3	3
x^{k}	$\sim$	$x^{44.5}$	3	2
$x^{44.5}$	$\sim$	$x^{49.125}$	2	2
$x^{49.125}$	$\sim$	x^{125}	2	1
x^{125}	$\sim$	$x^{141.75}$	1	1
$x^{141.75}$	$\sim$	x^{151}	1	0

The final region with the gauge symmetry G_{SM} is between $x^{151} M_1 (t = -35.00)$ and $x^{161.4} M_1 (= M_Z)$ $(t = -37.34)$, where the supersymmetry is broken. In this region, we have

$$
-b_i = \begin{pmatrix} -7 \\ -\frac{10}{3} + n_H \\ 4 + \frac{3}{5} n_H \end{pmatrix},
$$

$$
b_{ij} = \begin{pmatrix} -26 & 9/2 & 11/10 \\ 12 & 11/3 & 3/5 \\ 44/5 & 9/5 & 19/5 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 25/2 & 9/10 \\ 0 & 27/10 & 27/50 \end{pmatrix} n_H,
$$

$$
a_i = \begin{pmatrix} 2 \\ 3/2 \\ 17/10 \end{pmatrix} y^{(1U)}, \tag{60}
$$

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where

$$n_H = \begin{cases} 1 & x^{151} > Q/M_1 \geq x^{158}, \\ 0 & x^{158} > Q/M_1 \geq x^{161.4}. \end{cases} \tag{61}$$

The evolution equation for the Yukawa coupling is given by

$$\frac{dy^{(1U)}}{dt} = \frac{1}{2\pi} \left[-8\alpha_3 - \frac{9}{4}\alpha_{2L} - \frac{17}{20}\alpha_1 + \frac{9}{2}y^{(1U)} \right] y^{(1U)}. \tag{62}$$

We are now in a position to solve the RG equations numerically. By adjusting the relevant parameters, we can obtain some consistent solutions of the gauge unification. A typical solution is shown in Fig. 1, where the initial values of gauge coupling constants at the string scale M_S are taken as

$$\alpha_6(t = 0) = 0.057, \qquad \alpha_{2R}(t = 0) = 0.083 \tag{63}$$

and that of the Yukawa coupling as $y_0(t = 0) = 0.6$. In this solution the values of z_{33}, h_{33} and m_{33} in Eq. (13) are taken as 1.0, 0.3 and 2.0, respectively, and the parameter k is adjusted as $k = 42.5$. Under this choice of the parameter values, we obtain the running coupling constants at M_Z as

$$\alpha_3^{-1}(M_Z) = 8.44, \tag{64}$$
$$\alpha_{2L}^{-1}(M_Z) = 29.79, \tag{65}$$
$$\alpha_1^{-1}(M_Z) = 59.48. \tag{66}$$

These results are in good agreement with the experimental values.[23] Numerically, the results are insensitive to the values of z_{33}, h_{33} and m_{33} but appreciably affected by the value of k. For instance, if we take $k = 37.5$ instead of $k = 42.5$, the coupling constants are changed as

$$\alpha_3^{-1}(M_Z) = 8.26, \tag{67}$$
$$\alpha_{2L}^{-1}(M_Z) = 29.79, \tag{68}$$
$$\alpha_1^{-1}(M_Z) = 59.41. \tag{69}$$

In our model the spectra of the anti-generation fields $\overline{g}$ and $\overline{g}^c$ play an important role in adjusting the gauge couplings. Since the fields $\overline{g}$ and $\overline{g}^c$ are colored but $SU(2)_L$-singlets, $\alpha_3(M_Z)$ and $\alpha_1(M_Z)$ increase with decreasing k but $\alpha_{2L}(M_Z)$ is almost independent of k. We find a consistent solution of the gauge coupling unification, which
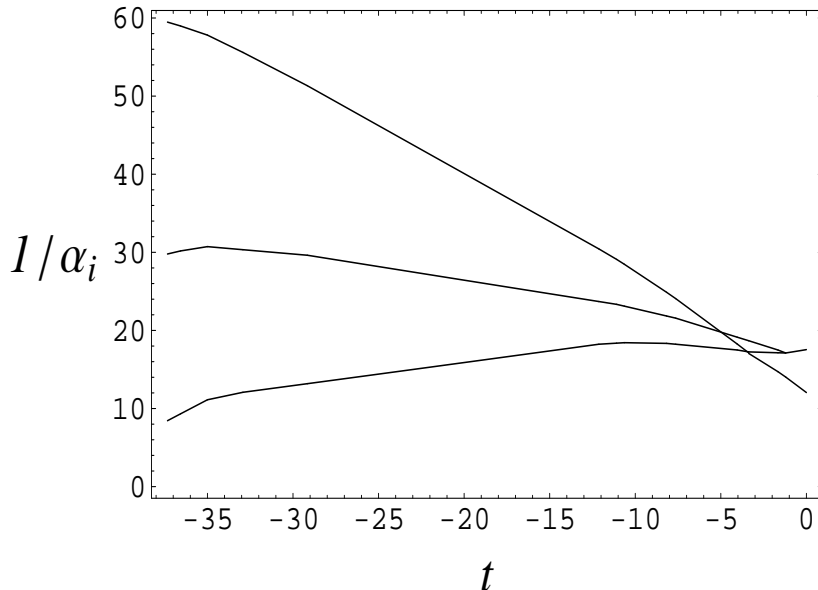
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Figure 1: Running inverse gauge couplings up to two-loop effects in our model. Vertical and horizontal axes show α_i^{-1} and $t = \ln(Q/M_S)$, respectively. The upper, middle and bottom lines in the left hand region correspond to α_1^{-1}, α_2^{-1} and α_3^{-1}, respectively. The initial values of Yukawa couplings are taken as $z_{33} = 1.0$, $h_{33} = 0.3$ and $m_{33} = 2.0$.

represents the connection between the string scale physics and the electroweak scale physics.

5 Summary and discussion

Characteristic patterns in fermion masses and mixings at low energies strongly suggest the existence of a profound type of the flavor symmetry including the R-parity. It is plausible that the flavor symmetry also controls the mass spectra of heavy particles which appear on the way between the string scale and the electroweak scale. In order to explore the path for connecting the string scale physics with the low-energy physics, it is necessary for us to study these spectra of heavy particles. For this purpose we examined particle spectra in the context of the $SU(6) \times SU(2)_R$ string-

inspired model with the flavor symmetry $\mathbf{Z}_{19} \times \mathbf{Z}_{18} \times \tilde{D}_4$. In our model the gauge symmetry is spontaneously broken in two steps as

$$SU(6) \times SU(2)_R \xrightarrow{\langle\phi_0\rangle} SU(4)_{\mathrm{PS}} \times SU(2)_L \times SU(2)_R \xrightarrow{\langle\psi_0\rangle} G_{\mathrm{SM}},$$

where $|\langle\phi_0\rangle| \sim 4.5 \times 10^{17}\,\mathrm{GeV}$ and $|\langle\psi_0\rangle| \sim 5.2 \times 10^{16}\,\mathrm{GeV}$. It was shown that there appear characteristic patterns of spectra also in the intermediate energy region. Afterward, we gave the two-loop RG equations of the gauge couplings in the intermediate region, in which each heavy particle decouples by its own stage. The RG runnings of the gauge couplings were studied up to two-loop order and we have obtained consistent gauge couplings at M_Z with the experimental values by taking the reasonable values for the available parameters. We explored solutions in which $SU(3)_c$ and $SU(2)_L$ gauge couplings meet at $\varnothing(5 \times 10^{17}\,\mathrm{GeV})$ and found a solution of the gauge coupling unification by adjusting the spectra of the anti-generation matter fields. The solution represents the connection between the string scale physics and the electroweak scale physics. It should be emphasized that in our model the unification scale is around the string scale ($\sim 10^{18}\,\mathrm{GeV}$) but not the so-called GUT scale ($\sim 2 \times 10^{16}\,\mathrm{GeV}$).

In the present analysis the gauge couplings at the string scale $M_S = 1.5 \times 10^{18}\,\mathrm{GeV}$ are numerically taken as

$$\alpha_6(t=0) = 0.057, \qquad \alpha_{2R}(t=0) = 0.083. \tag{70}$$

This means that the gauge couplings of $SU(6)$ and $SU(2)_R$ are not unified in the four-dimensional effective theory. In the framework of the higher-dimensional underlying theory, however, the gauge unification of $SU(6)$ and $SU(2)_R$ is expected to be realized non-perturbatively. There is a possibility that the $SU(6)$ and $SU(2)_R$ gauge groups live in distinct world-volumes of different D-branes. It has been pointed out that if the $SU(6)$ gauge group lives in the world-volume of 9-branes and the $SU(2)_R$ in the world-volume of 5-branes, or *vice versa*, unification solutions can be found in the vicinity of the self-dual point in the moduli space.[24]

In the present study we parameterized the magnitude of the effective Yukawa interactions $\overline{\phi}^3$ to adjust the spectra of the anti-generation fields. In order to determine the spectra of the anti-generation fields, we need to know the flavor charges of the Kähler class moduli fields. To this end, it is necessary to elucidate the origin of the flavor symmetry linked to the structure of the compact space. Interestingly, in the

case of the four-generation model we were able to obtain the flavor symmetry and to calculate the flavor charges of the matter superfields, explicitly.[7] In this case the compact space is a quintic hypersurface in CP^4. String compactification on this hypersurface corresponds to a deformation of the 3^5 Gepner model.[8] In addition, it is considered that the flavor symmetry has its origin not only in the symmetric structure of the compact space but also in the non-commutativity in the compact space.[1] In the context of M-theory on G_2 holonomy spaces, the gauge symmetry at the string scale is related to the singularity structure of the compact space[25] and constrained by the cohomology condition on the brane configuration.[26] In this context the flavor symmetry might be also constrained depending on the singularity structure of the compact space. The flavor symmetry is expected to provide an important clue to the study of the path for connecting the string scale physics with the low-energy physics.

Appendix

By using the effective superpotential Eq. (13), we can extract the relevant Yukawa terms which give the dominant contributions to the RG equations.

Due to the symmetry breaking of $SU(6) \times SU(2)_R$ down to $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$, the matter superfields $\phi(\mathbf{15}, \mathbf{1})$ and $\psi(\mathbf{6^*}, \mathbf{2})$ are decomposed as

$$
\begin{aligned}
\phi(\mathbf{15},\ \mathbf{1}) &\longrightarrow \phi(\mathbf{4},\ \mathbf{2},\ \mathbf{1}) + \phi(\mathbf{6},\ \mathbf{1},\ \mathbf{1}) + \phi(\mathbf{1},\ \mathbf{1},\ \mathbf{1}), \\
\psi(\mathbf{6^*},\ \mathbf{2}) &\longrightarrow \psi(\mathbf{4^*},\ \mathbf{1},\ \mathbf{2}) + \phi(\mathbf{1},\ \mathbf{2},\ \mathbf{2}),
\end{aligned}
\tag{A.1}
$$

respectively in the region between $x^{0.5} M_1$ and $x^{10.125} M_1$. Therefore, the effective superpotential is expressed as

$$
\begin{aligned}
W_Y^{(eff)} &= \frac{1}{2} f^{(\overline{Z})} \tilde{S}\, \overline{\phi(6,1,1)}^2 + \frac{1}{2} f^{(H_0)} \tilde{S}\, \psi(1,2,2)_0^2 \\[2mm]
&\quad + \frac{1}{2} f_{ij}^{(Z)} \tilde{S}\, \phi(6,1,1)_i\, \phi(6,1,1)_j + \frac{1}{2} f_{ij}^{(H)} \tilde{S}\, \psi(1,2,2)_i\, \psi(1,2,2)_j \\[2mm]
&\quad + M_{ij}^{(1)} \psi(1,2,2)_0\, \phi(4,2,1)_i\, \psi(4^*,1,2)_j \\
&\quad + M_{ij}^{(2)} \psi(4^*,1,2)_0\, \phi(4,2,1)_i\, \psi(1,2,2)_j \\
&\quad + M_{ij}^{(3)} \psi(4^*,1,2)_0\, \phi(6,1,1)_i\, \psi(4^*,1,2)_j
\end{aligned}
$$

$$+ M_{ij}^{(4)} \psi(1,2,2)_0 \, \phi(1,1,1)_i \, \psi(1,2,2)_j, \tag{A.2}$$

where the effective Yukawa couplings are of the forms

$$f^{(\overline{Z})} = \frac{(2\overline{\zeta}+1)}{\sqrt{2}} \, \overline{z} \, x^{\overline{\zeta}}, \qquad f^{(H_0)} = \frac{(2\eta_0+1)}{\sqrt{2}} \, h_0 \, x^{\eta_0},$$

$$f_{ij}^{(Z)} = \frac{(2\zeta_{ij}+1)}{\sqrt{2}} \, z_{ij} \, x^{\zeta_{ij}}, \qquad f_{ij}^{(H)} = \frac{(2\eta_{ij}+1)}{\sqrt{2}} \, h_{ij} \, x^{\eta_{ij}} \tag{A.3}$$

and

$$M_{ij}^{(a)} = m_{ij}^{(a)} \, x^{\mu_{ij}}. \qquad (a = 1,2,3,4) \tag{A.4}$$

As to the last equation, note that $M_{ij}^{(a)}(a = 1 \sim 4)$ evolves separately, but

$$M_{ij}^{(1)} = M_{ij}^{(2)} = M_{ij}^{(3)} = M_{ij}^{(4)} = m_{ij} \, x^{\mu_{ij}} \tag{A.5}$$

at scale $|\langle \phi_0 \rangle| = x^{0.5} M_1$. As seen in Eq. (8), the dominant contributions in the effective potential come from the terms with small powers of x. It turns out that such terms are the effective Yukawa interactions with $f_{33}^{(Z)}$, $f_{33}^{(H)}$ and $M_{33}^{(1\sim 4)}$ in Eq. (A.2). The RG equations for the Yukawa couplings are given by

$$\frac{dy^{(Z)}}{dt} = \frac{1}{2\pi} \left[-10\alpha_4 + 8y^{(Z)} + 2y^{(H)} + 4y^{(3)} \right] y^{(Z)},$$

$$\frac{dy^{(H)}}{dt} = \frac{1}{2\pi} \left[-3\alpha_{2L} - 3\alpha_{2R} + 6y^{(Z)} + 4y^{(H)} + 8y^{(2)} + 2y^{(4)} \right] y^{(H)},$$

$$\frac{dy^{(1)}}{dt} = \frac{1}{2\pi} \left[-\frac{15}{2}\alpha_4 - 3\alpha_{2L} - 3\alpha_{2R} + 8y^{(1)} + 2y^{(2)} + 3y^{(3)} + y^{(4)} \right] y^{(1)},$$

$$\frac{dy^{(2)}}{dt} = \frac{1}{2\pi} \left[-\frac{15}{2}\alpha_4 - 3\alpha_{2L} - 3\alpha_{2R} + y^{(H)} + 2y^{(1)} + 8y^{(2)} + 3y^{(3)} + y^{(4)} \right] y^{(2)},$$

$$\frac{dy^{(3)}}{dt} = \frac{1}{2\pi} \left[-\frac{25}{2}\alpha_4 - 3\alpha_{2R} + y^{(Z)} + 2y^{(1)} + 2y^{(2)} + 7y^{(3)} \right] y^{(3)},$$

$$\frac{dy^{(4)}}{dt} = \frac{1}{2\pi} \left[-3\alpha_{2L} - 3\alpha_{2R} + y^{(H)} + 4y^{(1)} + 4y^{(2)} + 6y^{(4)} \right] y^{(4)}. \tag{A.6}$$

In the region between $x^{10.125} M_1$ and $x^{151} M_1$, where the gauge group is $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$, the matter superfields are further decomposed as

$$
\begin{aligned}
\phi(\mathbf{4},\ \mathbf{2},\ \mathbf{1}) &\longrightarrow \phi(\mathbf{3},\ \mathbf{2},\ \mathbf{1/3}) + \phi(\mathbf{1},\ \mathbf{2},\ \mathbf{-1}), \\
\phi(\mathbf{6},\ \mathbf{1},\ \mathbf{1}) &\longrightarrow \phi(\mathbf{3},\ \mathbf{1},\ \mathbf{-2/3}) + \phi(\mathbf{3^*},\ \mathbf{1},\ \mathbf{2/3}), \\
\phi(\mathbf{1},\ \mathbf{1},\ \mathbf{1}) &\longrightarrow \phi(\mathbf{1},\ \mathbf{1},\ \mathbf{0}), \\
\psi(\mathbf{4^*},\ \mathbf{1},\ \mathbf{2}) &\longrightarrow \psi(\mathbf{3^*},\ \mathbf{1},\ \mathbf{-4/3}) + \phi(\mathbf{3^*},\ \mathbf{1},\ \mathbf{2/3}) \\
&\qquad\qquad + \psi(\mathbf{1},\ \mathbf{1},\ \mathbf{2}) + \phi(\mathbf{1},\ \mathbf{1},\ \mathbf{0}), \\
\psi(\mathbf{1},\ \mathbf{2},\ \mathbf{2}) &\longrightarrow \psi(\mathbf{1},\ \mathbf{2},\ \mathbf{1}) + \phi(\mathbf{1},\ \mathbf{2},\ \mathbf{-1}).
\end{aligned}
\tag{A.7}
$$

In this region we obtain five dominant Yukawa terms

$$
M_{33}^{(1U)}\ \tilde{H}_{u0}\ \tilde{Q}_3\ \tilde{U}_3^c, \qquad\qquad M_{33}^{(1D)}\ \tilde{H}_{d0}\ \tilde{Q}_3\ \tilde{g}_3^c,
$$

$$
\frac{1}{\sqrt{2}} M_{33}^{(3N)}\ \tilde{N}^c\ \tilde{g}_3\ \tilde{g}_3^c, \qquad\qquad M_{33}^{(3D)}\ \tilde{D}_0^c\ \tilde{g}_3\ \tilde{N}_3^c
$$

and

$$
f_{33}^{'(Z)}\ \tilde{S}\ \tilde{g}_3\ \tilde{g}_3^c = \left(x^{4.375} f_{33}^{(Z)} + \frac{1}{\sqrt{2}} M_{33}^{(3)} \right) \tilde{S}\ \tilde{g}_3\ \tilde{g}_3^c.
\tag{A.8}
$$

We use here the notation "$\sim$" for mass eigenstates. The last equation in the above comes from the g^c-D^c mixing. The RG equations for these Yukawa couplings are given by

$$
\frac{dy^{(1U)}}{dt} = \frac{1}{2\pi}\left[-\frac{16}{3}\alpha_3 - 3\alpha_{2L} - \frac{13}{15}\alpha_1 + 6y^{(1U)} + y^{(1D)} \right] y^{(1U)},
$$

$$
\frac{dy^{(1D)}}{dt} = \frac{1}{2\pi}\left[-\frac{16}{3}\alpha_3 - 3\alpha_{2L} - \frac{7}{15}\alpha_1 + y^{(1U)} + 6y^{(1D)} + \frac{1}{2}y^{(3N)} + y^{'(Z)} \right] y^{(1D)},
$$

$$
\frac{dy^{(3N)}}{dt} = \frac{1}{2\pi}\left[-\frac{16}{3}\alpha_3 - \frac{4}{15}\alpha_1 + 2y^{(1D)} + 5y^{(3N)} + y^{(3D)} + 2y^{'(Z)} \right] y^{(3N)},
$$

$$
\frac{dy^{(3D)}}{dt} = \frac{1}{2\pi}\left[-\frac{16}{3}\alpha_3 - \frac{4}{15}\alpha_1 + \frac{1}{2}y^{(3N)} + 5y^{(3D)} + y^{'(Z)} \right] y^{(3D)},
$$

$$
\frac{dy^{'(Z)}}{dt} = \frac{1}{2\pi}\left[-\frac{16}{3}\alpha_3 - \frac{4}{15}\alpha_1 + 2y^{(1D)} + y^{(3N)} + y^{(3D)} + 5y^{'(Z)} \right] y^{'(Z)}.
\tag{A.9}
$$

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Acknowledgements

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